Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

**Time** 2 hours

**Paper reference** **8MA0/01**

# Mathematics

## Advanced Subsidiary
## PAPER 1: Pure Mathematics

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

*Turn over*

**1.** **In this question you should show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

**(3)**

subtract 20 from both sides to get it into a quadratic form

$$x^2 - x > 20$$

$$x^2 - x - 20 > 0$$

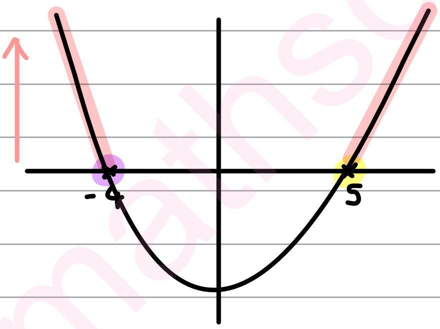
factorise:

$$(x+4)(x-5) > 0$$

find critical values:

$$x = -4 \qquad x = 5$$

sketch a graph:




we need to find values greater than 0, so look for values above the x axis.

$$x < -4 \qquad x > 5$$

use curly brackets because it's a set

: means such that

the question requires set notation

$$\{x : x < -4\} \cup \{x : x > 5\}$$

means or

ie. the set of $x$ values such that $x$ is less than $-4$.

$$\boxed{\{x : x < -4\} \cup \{x : x > 5\}}$$





P 6 6 5 8 5 A 0 2 4 4

**Question 1 continued**

**(Total for Question 1 is 3 marks)**

2. **In this question you should show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express $y$ in terms of $x$, writing your answer in simplest form.

**(3)**

9 and 81 are both powers of 3, so they can be written as $3^a$

$3 = 3^1$
$9 = 3^2$
$81 = 3^4$

$$\frac{(3^2)^{x-1}}{(3^1)^{y+2}} = 3^4$$

now use rule: $\boxed{(x^a)^b = (x^{ab})}$

This means we can multiply the powers together, so $(3^2)^{x-1} = 3^{2x-2}$

$$\frac{3^{2(x-1)}}{3^{y+2}} = 3^4$$

$$\frac{3^{2x-2}}{3^{y+2}} = 3^4$$

multiply both sides by $3^{y+2}$

$$3^{2x-2} = (3^4)(3^{y+2})$$

$\boxed{(x^a)(x^b) = x^{a+b}}$

This means if the base is the same and we multiply them together, we add the powers

simplify the right hand side

$$3^{2x-2} = 3^{4+y+2}$$

$$3^{2x-2} = 3^{y+6}$$

we can compare the powers, because there is a single power of 3 on each side. Therefore, the powers must be equal

$$2x-2 = y+6$$

$$2x-8 = y$$

simplify to make y the subject.

$$\boxed{y = 2x-8}$$



P 6 6 5 8 5 A 0 4 4 4

**Question 2 continued**

**(Total for Question 2 is 3 marks)**

3. Find

$$\int \frac{3x^4 - 4}{2x^3}\,dx$$

writing your answer in simplest form.

**(4)**

The fraction can be separated to make terms which you can easily integrate

$$\int \frac{3x^4-4}{2x^3}\,dx$$

by separating it into two fractions with the same denominator, we can simplify each fraction in order to integrate it.

$$= \int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$$

$\frac{x^4}{x^3} = x^1$ because when we divide by a term with the same base, we subtract the bottom power from the top

$$= \int \frac{3}{2}x - 2x^{-3}\,dx$$

$-\frac{4}{2} = -2$, $\frac{1}{x^3} = x^{-3}$, because 1 divided by $x^a = x^{-a}$

so $-\frac{4}{2x^3} = 2x^{-3}$

integrate, by adding 1 to the power, and dividing by the new power

$$= \frac{3x^2}{2\times 2} - \frac{2x^{-2}}{-2} + c$$

$$= \frac{3}{4}x^2 - \frac{2}{-2}x^{-2} + c$$

simplify:

$$= \frac{3}{4}x^2 + x^{-2} + c$$

Can also be written as: because $x^{-2} = \frac{1}{x^2}$

$$\boxed{\frac{3}{4}x^2 + \frac{1}{x^2} + c}$$

remember to add c, because we are integrating without limits.

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**Question 3 continued**

**(Total for Question 3 is 4 marks)**

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point $O$.

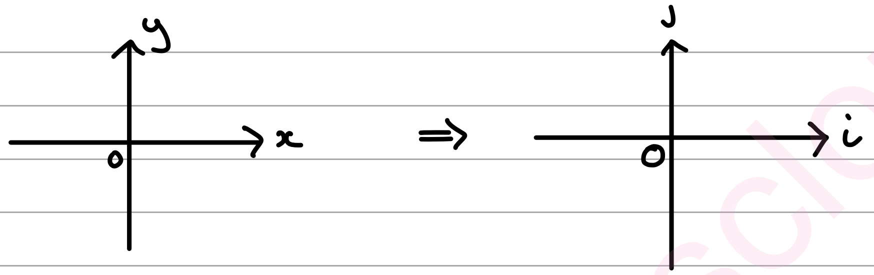
After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point $O$.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

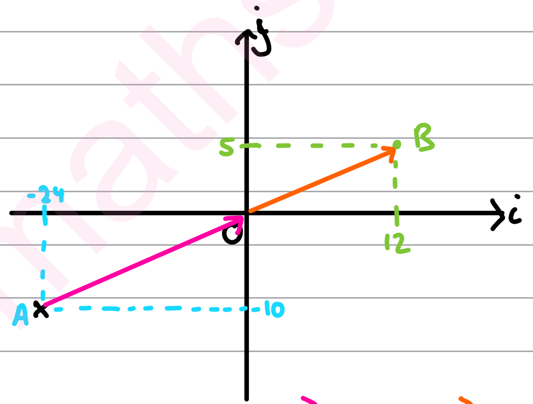
Using the model,

(a) prove that the stone passes through $O$, **(2)**

(b) calculate the speed of the stone. **(3)**




Draw A and B on a diagram




to prove the stone passes through O, find $\overrightarrow{AO}$ and $\overrightarrow{OB}$. If $\overrightarrow{AO}$ is a multiple of $\overrightarrow{OB}$, we know that both $\overrightarrow{AO}$ and $\overrightarrow{OB}$ lie on the same line which must pass through O.

Find $\overrightarrow{AO}$ by subtracting A from O, as we always subtract the first letter from the 2nd.

$$\overrightarrow{AO} = O - A = (0i + 0j) - (-24i - 10j)$$
$$= 0i + 0j + 24i + 10j$$
$$\overrightarrow{AO} = 24i + 10j$$

Find $\overrightarrow{OB}$ by subtracting O from B.

**Question 4 continued**

$\vec{OB} = B - O = (12i + 5j) - (0i + 0j)$

$= 12i + 5j - 0i - 0j$

$\vec{OB} = 12i + 5j$

this is just the same as vector B because vector B is relative to O.

now compare $\vec{AO}$ and $\vec{OB}$

$24i + 10j = 2(12i + 5j)$

$\vec{AO} = 2\vec{OB}$

Because $\vec{AO}$ is a multiple of $\vec{OB}$, $\vec{AO}$ and $\vec{OB}$ are parallel. Because the stone is travelling in a straight line, it must pass through O, because $\vec{AO}$ and $\vec{OB}$ lie on the same line

b) we can work out speed using $\boxed{\text{speed} = \frac{\text{distance}}{\text{time}}}$

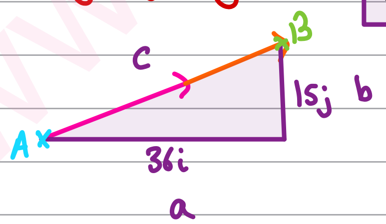
we know $t = 4$, so find distance $|\vec{AB}|$

$\vec{AB} = \vec{AO} + \vec{OB}$

$= 24i + 10j + 12i + 5j$

$\vec{AB} = 36i + 15j$

to find the distance, find the magnitude of $\vec{AB}$, $|\vec{AB}|$. Do this by using pythagoras. $\boxed{a^2 + b^2 = c^2}$




$c^2 = a^2 + b^2$

$\vec{AB} = c = \sqrt{a^2 + b^2}$

$= \sqrt{36^2 + 15^2}$

$= \sqrt{1521}$

$\vec{AB} = 39 = \text{distance}$

$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{39}{4} = \boxed{9.75\text{ms}^{-1}}$

**(Total for Question 4 is 5 marks)**

**5.**

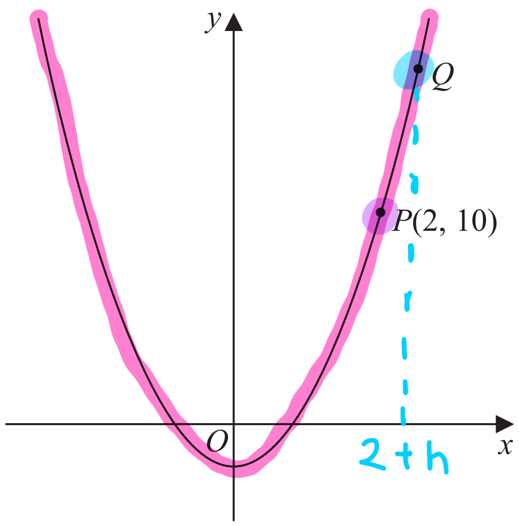



**Figure 1**

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at $P$.

**(2)**

The point $Q$ with $x$ coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line $PQ$, giving your answer in terms of $h$ in simplest form.

**(3)**

(c) Explain briefly the relationship between part (b) and the answer to part (a).

**(1)**

a) To find the gradient of the tangent at (2,10), find $\frac{dy}{dx}$, and sub in the $x$ value, $x=2$

to differentiate, multiply by the power, and subtract 1 from the power.

$$y = 3x^2 - 2$$

$$\frac{dy}{dx} = 6x$$

Sub in the $x$ co-ordinate of P (2) to find gradient of tangent at P

@ $x=2$ $\quad \frac{dy}{dx} = 6 \times 2$

$= 12$

$\boxed{\text{gradient of tangent at P is 12}}$

**Question 5 continued**

b) find the y value of Q by subbing $x = 2+h$ into $y = 3x^2 - 2$

$y_q = 3(2+h)^2 - 2$

now find the gradient of PQ by using $\frac{y_q - y_p}{x_q - x_p}$

this is how we find a gradient when finding the equation of a straight line.

$P(2, 10) \rightarrow x_p = 2 \quad y_p = 10$

$Q(2+h, 3(2+h)^2 - 2) \rightarrow x_q = 2+h \quad y_q = 3(2+h)^2 - 2$

$$\text{gradient} = \frac{y_q - y_p}{x_q - x_p} = \frac{(3(2+h)^2 - 2) - 10}{(2+h) - 2}$$

simplify to get in its simplest form

$$= \frac{3(2+h)^2 - 2 - 10}{(2+h) - 2}$$

$$= \frac{3(4 + 4h + h^2) - 12}{h}$$

$$= \frac{12 + 12h + 3h^2 - 12}{h}$$

$$= \frac{12h + 3h^2}{h}$$

$$\boxed{\text{gradient} = 12 + 3h}$$

tends to: gets very close to

c) As h tends to 0, 12 + 3h tends to 12. So the gradient of the chord PQ tends to the gradient of the tangent of the curve

**(Total for Question 5 is 6 marks)**

6. **In this question you should show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

**(3)**

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

**(3)**

a) $3x^3 - 17x^2 - 6x = 0$ is a cubic, so we cannot solve by factorising normally or using the quadratic formula.

All elements contain $x$, so we can factorise out $x$

$3x^3 - 17x^2 - 6x = 0$

$x(3x^2 - 17x - 6) = 0$

now factorise the quadratic

$x(3x+1)(x-6) = 0$

now find all values of $x$ by making each bracket equal to 0.

$x = 0$    $3x + 1 = 0$, $3x = -1$, $x = -1/3$    $x - 6 = 0$, $x = 6$

$\boxed{x = 0, -1/3, 6}$

b) $3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$

To compare the two equations, look at what is multiplied by -6. In the first equation it's $x$, and in the second it's $(y-2)^2$, so we can see $x = (y-2)^2$

$-6(y-2)^2 = -6x$

$(y-2)^2 = x$

Now use all values of $x$ we found in part a to find the corresponding $y$ values

① $x = 0$    $0 = (y-2)^2$

$0 = y - 2$

$y = 2$



P 6 6 5 8 5 A 0 1 2 4 4

**Question 6 continued**

② $x = -\frac{1}{3}$ $\quad -\frac{1}{3} = (y-2)^2$ $\quad \swarrow$ we cannot take the square root of a negative number, so $x = -\frac{1}{3}$ does not give any real solutions of $y$.

③ $x = 6$ $\quad 6 = (y-2)^2$

$\sqrt{6} = y - 2$

$y = \sqrt{6} + 2$

$$\boxed{y = 2, \quad y = \sqrt{6} + 2}$$

**(Total for Question 6 is 6 marks)**

**7.** A parallelogram $PQRS$ has area 50 cm$^2$

has two sets of parallel sides

Given

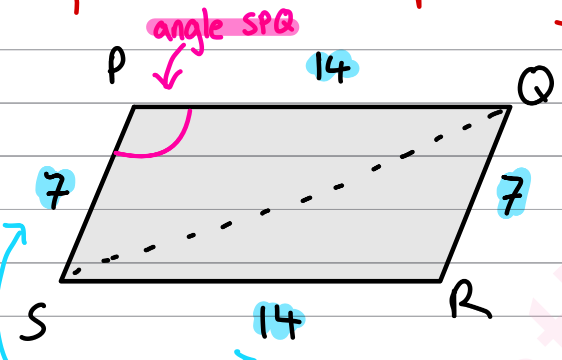
- $PQ$ has length 14 cm
- $QR$ has length 7 cm
- angle $SPQ$ is obtuse

find

(a) the size of angle $SPQ$, in degrees, to 2 decimal places, **(3)**

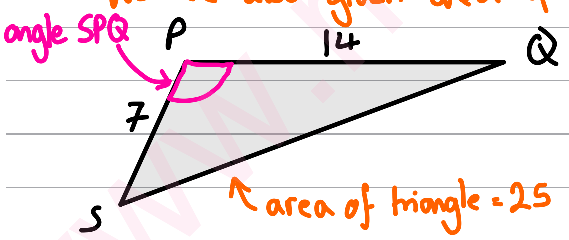
(b) the length of the diagonal $SQ$, in cm, to one decimal place. **(2)**

a) First, draw a sketch representing the information given




PS and SR equal 7 and 14 respectively because on a parallelogram, the parallel sides are equal in length

We are also given area of parallelogram = 50




If we split the parallelogram into two triangles with the same area, we can work out the size of angle SPQ

area of triangle = $\frac{50}{2} = 25$

$\boxed{\text{area of a triangle} = \frac{1}{2}ab\sin C}$

where C is the angle we are trying find (SPQ) and a and b are the known lengths

$25 = \frac{1}{2} \times 7 \times 14 \times \sin C$



P 6 6 5 8 5 A 0 1 4 4 4

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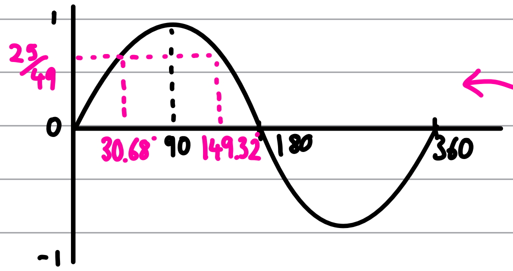
**Question 7 continued**

$$25 = 49 \sin C$$
$$25/49 = \sin C$$
$$\sin^{-1} 25/49 = C$$
$$C = 30.68^\circ$$

$C = 30.7^\circ$. However, in the question, we are told angle SPQ is obtuse. This means it must be greater than 90°

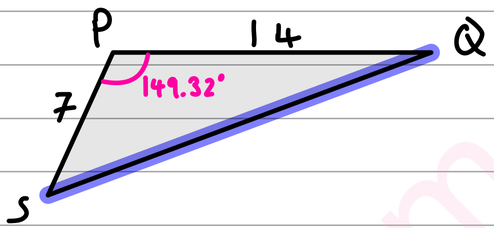



Find angle SPQ by subtracting 30.7° from 180°, or by looking at the sin graph to find another solution for $25/49 = \sin C$

$$180 - 30.68^\circ = 149.32^\circ$$

$$\boxed{\text{angle } SPQ = 149.32^\circ}$$

b)




To find an unknown length of a scalene triangle, use $\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

unknown length $a = SQ$

known lengths

known angle SPQ

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 7^2 + 14^2 - 2(7)(14)\cos(149.32)$$
$$a^2 = 245 - 196\cos(149.32)$$
$$a^2 = 245 - (196 \times -0.860)$$

$\cos 149 = -0.86$

$$a^2 = 245 + 168.6$$
$$a^2 = 413.6$$
$$a = \sqrt{413.6}$$
$$= 20.336..$$
$$a = 20.3 \text{ (1dp)}$$

$a = SQ$ so $\boxed{SQ = 20.3\text{cm}}$

**(Total for Question 7 is 5 marks)**

**8.** $\mathrm{g}(x) = (2 + ax)^8$ where $a$ is a constant

Given that one of the terms in the binomial expansion of $\mathrm{g}(x)$ is $3402x^5$

power of $x$ is 5

(a) find the value of $a$. **(4)**

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

Using this value of $a$,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

**(3)**

a) Expand $(2+ax)^8$ up to $x^5$, to find the coefficient of $x^5$.

you can use pascal's triangle, or use the nCr function on your calculator

$$g(x) = 1\times 2^0 \times (ax)^8 + 8\times 2^1\times(ax)^7 + 28\times 2^2\times(ax)^6 + 56\times 2^3\times(ax)^5 \ldots\ldots$$

$$\boxed{(ab)^x = a^x b^x}$$

this is the term we need because the power of $x$ is 5, so we can stop expanding

$$56\times 2^3\times(ax)^5 = 3402x^5$$

$$56\times 8\times a^5x^5 = 3402x^5$$

$x^5$ is on both sides, so divide both sides by $x^5$ to solve for a.

$$56\times 8\times a^5 = 3402$$

$$448a^5 = 3402$$

$$a^5 = \frac{3402}{448}$$

$$a = \sqrt[5]{\frac{2402}{448}}$$

$$\boxed{a = \frac{3}{2}}$$

**Question 8 continued** $\left(1 + \frac{1}{x^4}\right)\left(2 + ax\right)^8$

b) The constant term is where the power of $x$ is 0, this happens twice here.

1. When 1 is multiplied by the constant term in the expansion of the 2nd bracket
2. When $1/x^4$ is multiplied by the $x^4$ term in the expansion of the 2nd bracket

So, we need to find the constant term, and the $x^4$ term in the second bracket using the binomial expansion.

constant term: $1 \times 2^8 \times \left(\frac{3}{2}x\right)^0$

$= 2^8$

$= 256$

$x^4$ term $= 70 \times 2^4 \times \left(\frac{3}{2}x\right)^4$

$= 70 \times 16 \times \frac{81}{16}x^4$

$= 5670x^4$

| | $256$ | $5670x^4$ |
|---|---|---|
| $1$ | $\boxed{256}$ | $5670x^4$ |
| $\frac{1}{x^4}$ | $\frac{256}{x^4}$ | $\boxed{5670}$ |

these are the two terms we need because they are both constant.

To find the final constant term, just add the two constant terms we have found together

$256 + 5670 = 5926$

$\boxed{\text{constant term} = 5926}$

**(Total for Question 8 is 7 marks)**

**9.** Find the value of the constant $k$, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

**(4)**

Rewrite the fraction so it is easy to integrate

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

$\sqrt{x} = x^{1/2}$

$$\int_k^9 \frac{6}{x^{1/2}}\,dx = 20$$

$\frac{1}{x^a} = x^{-a}$

$\therefore \frac{1}{x^{1/2}} = x^{-1/2}$

$$\int_k^9 6x^{-1/2}\,dx = 20$$

Now integrate by adding 1 to the power, and dividing by the new power.

$-\frac{1}{2} + 1 = \frac{1}{2} \rightarrow$ $= \left[\frac{6}{\frac{1}{2}} x^{1/2}\right]_k^9 = 20$ ← we don't need to add C because we are integrating with limits.

$\frac{6}{\frac{1}{2}} = 6 \times 2 = 12 \rightarrow$ $= \left[12\sqrt{x}\right]_k^9 = 20$

Sub in 9 and k $\quad (12\sqrt{9}) - (12\sqrt{k}) = 20$

Simplify and rearrange to find k

$$(12 \times 3) - 12\sqrt{k} = 20$$

$$36 - 12\sqrt{k} = 20$$

$$16 - 12\sqrt{k} = 0$$

$$16 = 12\sqrt{k}$$

$$4/3 = \sqrt{k}$$

$$\boxed{k = \frac{16}{9}}$$

**Question 9 continued**

**(Total for Question 9 is 4 marks)**

**10.** A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

**(4)**

(b) Use a counterexample to show that the statement is not always true.

**(1)**

a) When we are doing a proof with odd numbers, odd numbers are represented by $2x+1$.

because we are showing that it is true for odd numbers, let $n = 2x+1$

sub in $n=2x+1$

$n^3 - n$

$= (2x+1)^3 - (2x+1)$

expand by multiplying out one bracket at a time, or by using binomial expansion.

$= (4x^2+4x+1)(2x+1) - 2x - 1$

$= (8x^3 + 8x^2 + 2x + 4x^2 + 4x + 1) - 2x - 1$

$= 8x^3 + 12x^2 + 4x$

$= 4(2x^3 + 3x + 1)$

we are trying to show that it is a multiple of 4, so if we can take out a factor of 4, that means it is a multiple of 4.

$\therefore$ $n^3 - n$ is a multiple of 4 when n is odd, because $4(2x^3+3x+1)$ is a multiple of 4.

b) Use trial and error to find an even value of n in which $n^3 - n$ is not a multiple of 4.

when $n=2$ $n^3 - n$

$= 2^3 - 2$

$= 8 - 2$

$= 6$ which is not a multiple of 4

so $n^3 - n$ is not always true.

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**Question 10 continued**

**Question 10 continued**

**Question 10 continued**

**(Total for Question 10 is 5 marks)**

**11.** The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, $A$ km$^2$, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where $c$ is a constant and $t$ is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

**(1)**

On 1st January 2019 an area of 60 km$^2$ of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of $c$ to 3 significant figures.

**(4)**

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km$^2$ of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

**(1)**

a) Just before tree planting was started, $t=0$. So sub in $t=0$ into $A = 80 - 45e^{ct}$ to find $A$.

@$t=0$ $\quad A = 80 - 45e^{0c}$ $\quad$ $e^0 = 1$ because anything to the power of 0 $= 1$.

$= 80 - 45$

$\boxed{= 35 \text{ km}^2}$

b) $A = 80 - 45e^{ct}$

The 1st January 2019 is 14 years after 1st January 2005 so $t = 14$.

We are given $A = 60$

Sub in $A=60$ and $t = 14$ to solve for $c$

$60 = 80 - 45e^{14c}$

$-20 = -45e^{14c}$

$20 = 45e^{14c}$

$4/9 = e^{14c}$

Question 11 continued

$\ln 4/9 = \ln e^{14c}$

$\ln 4/9 = 14c$

$\boxed{\ln e^{x} = x \quad \text{so } \ln e^{14c} = 14c}$

$c = \frac{1}{14} \ln 4/9$ ← put into your calculator

$c = -0.0579$ (to 3sf)

sub c back into the equation to find the complete model

$$\boxed{A = 80 - 45e^{-0.0579t}}$$

(Total for Question 11 is 6 marks)

**12.** **In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

**(5)**

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

1. $3\tan x - 5\sin x = 0$
2. $3\dfrac{\sin x}{\cos x} - 5\sin x = 0$
3. $3\sin x - 5\sin x\cos x = 0$
4. $3 - 5\cos x = 0$
5. $\cos x = \dfrac{3}{5}$
6. $x = 53.1°$

Identify two errors or omissions made by this student, giving a brief explanation of each.

**(2)**

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$

(b) Find, to the nearest degree, the value of $\alpha_4$

**(2)**

i) $\cos^2\theta$ can be written in terms of $\sin\theta$.

$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$

So, we can make the equation in terms of sin only.





P 6 6 5 8 5 A 0 2 6 4 4

**Question 12 continued**

$\cos^2\theta = 1 - \sin^2\theta$

$$5\cos^2\theta = 6\sin\theta$$
$$5(1-\sin^2\theta) = 6\sin\theta$$
$$5 - 5\sin^2\theta = 6\sin\theta$$

rearrange into a quadratic equation in terms of $\sin\theta$.

a quadratic equation in terms of $\sin\theta$, so we can solve using the quadratic equation,

$$0 = \underset{5=a}{5}\sin^2\theta + \underset{6=b}{6}\sin\theta \underset{c=-5}{-5}$$

$$\sin\theta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\sin\theta = \frac{-6 \pm \sqrt{6^2 - 4\times5\times-5}}{2\times5}$$

$$\sin\theta = \frac{-6 \pm \sqrt{36+100}}{10}$$

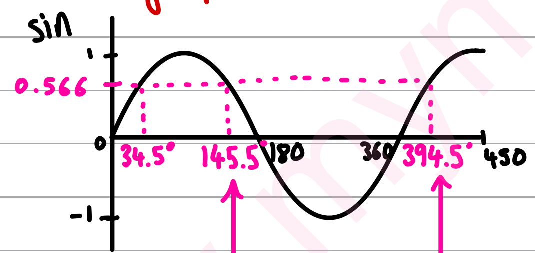
$$\sin\theta = \frac{-6 \pm \sqrt{136}}{10}$$

$$\sin\theta = 0.566, \cancel{-1.766}$$

$\sin\theta \neq -1.766$ because $\sin\theta$ must always be between $-1$ and $1$

$$\theta = 34.5^\circ$$

the range is $0 < \theta \leqslant 450^\circ$, so find the remaining values by sketching a sin graph.




$180 - 34.5 = 145.5^\circ$

$34.5 + 180 = 394.5^\circ$

$$\boxed{\theta = 34.5^\circ, 145.5^\circ, 394.5^\circ}$$

ii a) 1. Between lines 3 and 4, the student cancels out $\sin x$. This means they miss the solution $\sin x = 0 \therefore x = 0$. (The student should have factorised $\sin x$ out so that they could find $\sin x = 0$.)

2. The student did not find all solutions in the given range $-90^\circ < x < 90$ (in lines 5 and 6). Because the cos graph is symmetrical, they missed the solution $x = -53.1$



Turn over

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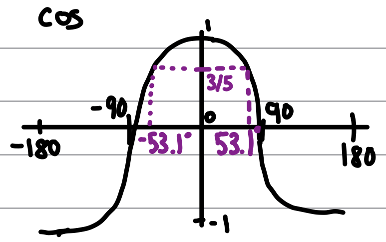
**Question 12 continued**



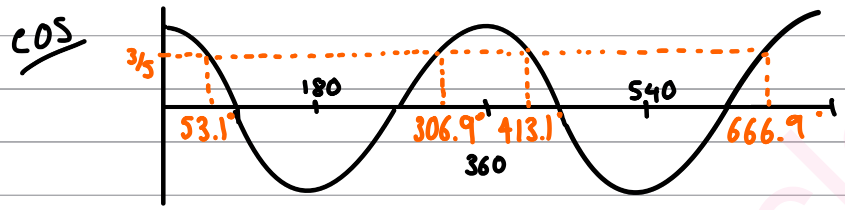

drawing the graph is not required, just shows where the other solution is.

b) $\cos(5\alpha + 40) = 3/5$

$(5\alpha + 40) = \cos^{-1}(3/5)$

$5\alpha + 40 = 53.1$

We need to find the 4th value, so sketch the cos graph and find the 4th point where $\cos(5\alpha + 40) = 3/5$




$5\alpha + 40 = 53.1°, 306.9°, 413.1°, \underline{666.9°}$

this is the value of $5\alpha+40$ we need because we are looking for $\alpha_4$.

4th value:

$666.9 = 5\alpha + 40$

$626.9 = 5\alpha$

$\alpha_4 = 125.4$

$\boxed{\alpha_4 = 125° \text{ to the nearest degree}}$

**Question 12 continued**

**(Total for Question 12 is 9 marks)**

**13.**

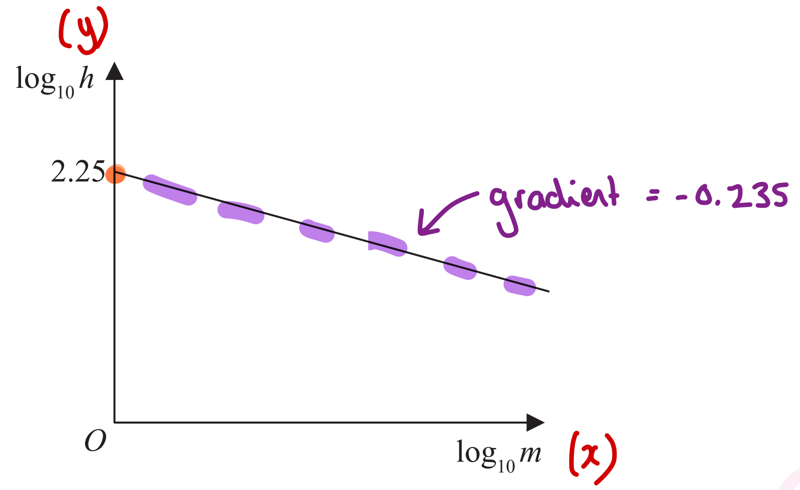



**Figure 2**

The resting heart rate, $h$, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where $p$ and $q$ are constants and $m$ is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of $-0.235$

(a) Find, to 3 significant figures, the value of $p$ and the value of $q$.

**(3)**

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

**(3)**

(c) With reference to the model, interpret the value of the constant $p$.

**(1)**

a) We can use the linear model $y = mx + c$, where $y = \log_{10} h$ and $x = \log_{10} m$

gradient = $-0.235$
y intercept = $2.25$

$$y = mx + c$$

$$\log_{10} h = -0.235 \log_{10} m + 2.25$$

raise each side to the power of 10.
$10^{\log_{10} h} = h$
so the left hand side is now equal to $h$

$$\log_{10} h = 2.25 - 0.235 \log_{10} m$$

$$h = 10^{2.25 - 0.235 \log_{10} m}$$

**Question 13 continued**

$h = 10^{2.25} \times 10^{-0.235 \log_{10} m}$

use rule $a \log x = \log x^a$

$-0.235 \log_{10} m = \log_{10} m^{-0.235}$

$h = 10^{2.25} \times 10^{\log_{10} m^{-0.235}}$

$\boxed{10^{\log_{10} x} = x}$

$h = 10^{2.25} \times m^{-0.235}$

so $10^{\log_{10} m^{-0.235}} = m^{-0.235}$

$h = 178 m^{-0.235}$

$\boxed{\text{So } p = 178 \text{ and } q = -0.235 \text{ to 3sf}}$

b) Sub in $m = 5$ and see what value of h the formula gives

$h = 178 m^{-0.235}$
$h = 178 \times 5^{-0.235}$
$= 178 \times 0.685$
$= 121.9$ bpm
$= 122$ bpm (3sf)

now compare this to the actual value of h given in the question,

The actual value of h is 119bpm. This is very close to 122bpm, so the model is suitable.

c) $h = pm^q$

p would be the resting heart rate of a mammal with a mass of 1kg. (This is because when $m = 1$, $m^{-0.235} = 1$ also, so $h = p$ when $m = 1$)

**Question 13 continued**

**Question 13 continued**

**(Total for Question 13 is 7 marks)**

**14.** A curve $C$ has equation $y = \text{f}(x)$ where

$$\text{f}(x) = -3x^2 + 12x + 8$$

(a) Write f($x$) in the form

$$a(x + b)^2 + c$$

where $a$, $b$ and $c$ are constants to be found.

**(3)**

The curve $C$ has a maximum turning point at $M$.

(b) Find the coordinates of $M$.

**(2)**

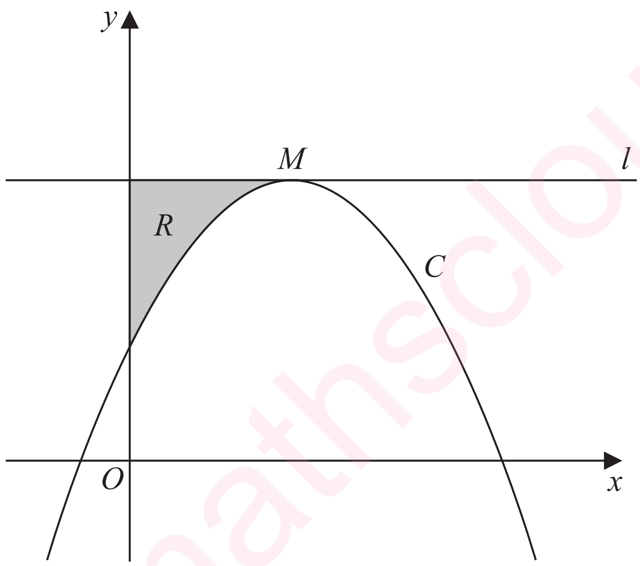



**Figure 3**

Figure 3 shows a sketch of the curve $C$.

The line $l$ passes through $M$ and is parallel to the $x$-axis.

The region $R$, shown shaded in Figure 3, is bounded by $C$, $l$ and the $y$-axis.

(c) Using algebraic integration, find the area of $R$.

**(5)**

a) Complete the square of $f(x) = -3x^2 + 12x + 8$

$$f(x) = -3x^2 + 12x + 8$$
take out a factor of 3 from the first two terms
$$= -3(x^2 - 4x) + 8$$
then complete the square inside the brackets
$$= -3((x-2)^2 - 4) + 8$$
remember to multiply the $-4$ by $-3$
$$= -3(x-2)^2 + 12 + 8$$

$$\boxed{f(x) = -3(x-2)^2 + 20}$$



P 6 6 5 8 5 A 0 3 4 4 4

**Question 14 continued**

b) The curve is in the form $a(x+b)^2+c$. The co-ordinates of the turning points are always $(-b, c)$

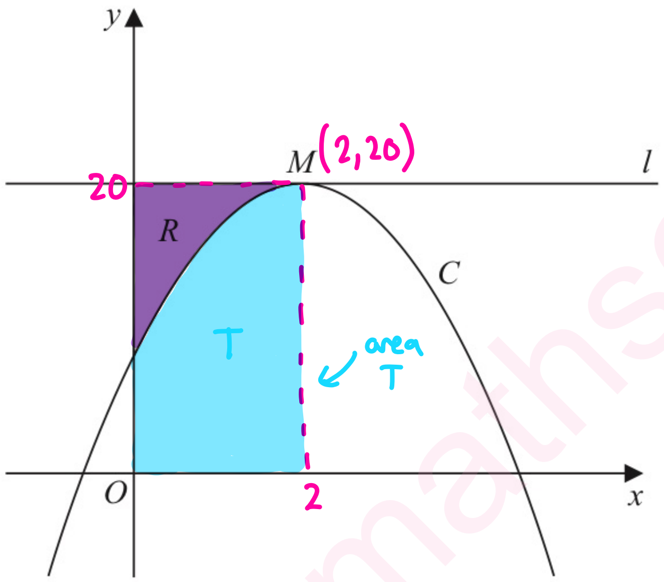
$f(x) = -3(x-2)^2 + 20$

$b = -2$ $\quad$ $c = 20$

$(-b, c) = (2, 20)$

$\boxed{\text{co-ordinates of } M = (2, 20)}$

c)




To find the area of R, we need to integrate curve C to find area T, and subtract it from the rectangle which contains R and T.

area of rectangle R + T = $20 \times 2 = 40$

area of R $= 40 -$ area of T

$= 40 - \int_0^2 -3x^2 + 12x + 8 \, dx$

$= 40 - \left[ -\frac{3}{3}x^3 + \frac{12}{2}x^2 + 8x \right]_0^2$

$= 40 - \left[ -x^3 + 6x^2 + 8x \right]_0^2$

integrate to find the area by adding 1 to the power and dividing by the new power

$= 40 - \left( (-2^3 + 6 \times 2^2 + 8 \times 2) - (-0^3 + 6 \times 0^2 + 8 \times 0) \right)$

**Question 14 continued**

$$= 40 - \left(\left(-8 + 24 + 16\right) - 0\right)$$

$$= 40 - (32)$$

$\boxed{\text{area of } R = 8}$

**Question 14 continued**

**(Total for Question 14 is 10 marks)**

**15.**

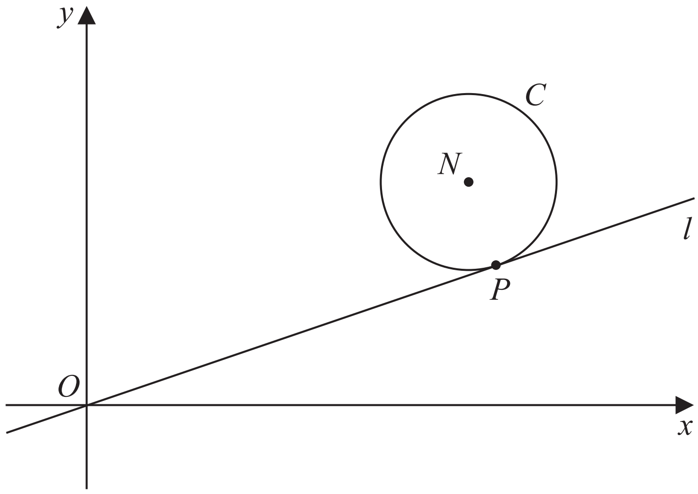



**Figure 4**

Figure 4 shows a sketch of a circle $C$ with centre $N(7, 4)$

The line $l$ with equation $y = \frac{1}{3}x$ is a tangent to $C$ at the point $P$.

Find

(a) the equation of line $PN$ in the form $y = mx + c$, where $m$ and $c$ are constants,

**(2)**

(b) an equation for $C$.

**(4)**

The line with equation $y = \frac{1}{3}x + k$, where $k$ is a non-zero constant, is also a tangent to $C$.

(c) Find the value of $k$.

**(3)**

a) To find PN, we need to know the gradient and a point on the line to sub into $y = mx + c$

the tangent is always perpendicular to the line from the centre of the circle to the point where the tangent touches the circle.

gradient of $l$: $\frac{1}{3}$  gradient of PN $= -3$  because $\frac{1}{3} \times -3 = -1$

so their two gradients multiply to give $-1$

point N = (7, 4)

$y = mx + c$

$y = -3x + c$

$4 = -3 \times 7 + c$

$4 = -21 + c$

$c = 25$

$\boxed{y = -3x + 25}$

b) We already have the centre of C so we need to find the radius

P is where the tangent and circle meet, so solve $y = -3x + 25$ and $y = \frac{1}{3}x$ simultaneously to find co-ordinates of P.

$y = \frac{1}{3}x$

$y = -3x + 25$

set equal $\rightarrow$

$\frac{1}{3}x = -3x + 25$

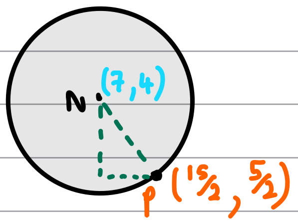
$\frac{10}{3}x = 25$

$10x = 75$

$x = \frac{15}{2}$

$y = \frac{1}{3}x$, so $y = \frac{5}{2}$

$P = \left(\frac{15}{2}, \frac{5}{2}\right)$




Find length PN using pythagoras, which is the radius.

$$\boxed{a = \sqrt{b^2 + c^2}}$$

$$r = PN = \sqrt{\left(7 - \frac{15}{2}\right)^2 + \left(4 - \frac{5}{2}\right)^2}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$$

$$r = PN = \sqrt{\frac{5}{2}}$$

the equation of a circle is $(x-a)^2 + (y-b)^2 = r^2$, where $(a,b)$ are the co-ordinates of the centre $(7,4)$ and r is the radius, $\sqrt{\frac{5}{2}}$

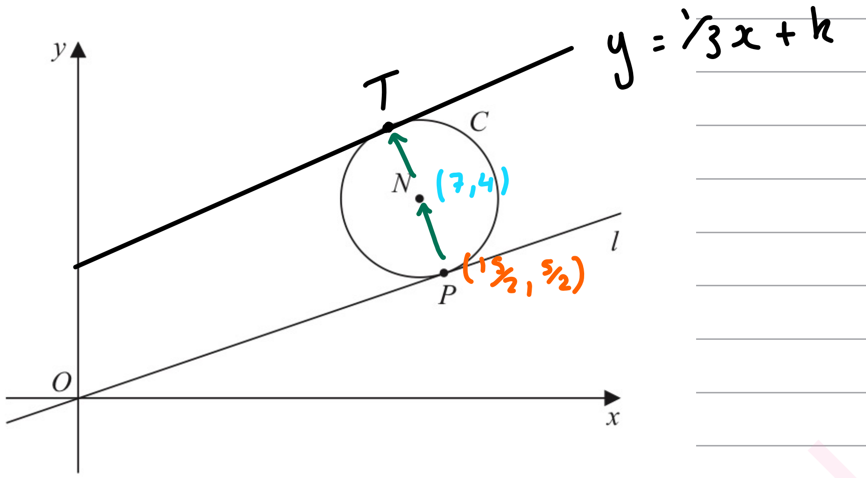
$$(x-a)^2 + (y-b)^2 = r^2$$

$$(x-7)^2 + (y-4)^2 = \sqrt{\frac{5}{2}}^{\,2}$$

$$\boxed{(x-7)^2 + (y-4)^2 = \frac{5}{2}}$$

**Question 15 continued**

c)




(T is the circle and the line's point of intersection)

We want to find the co-ordinates of T, so we can sub the values into $y = \frac{1}{3}x + k$ to find the value of k

We know T and P are equidistant from N because they lie on the radius, so $\overrightarrow{PN} = \overrightarrow{NT}$

$\overrightarrow{PN} = N - P$ (since we always subtract the first letter from the 2nd)

$= (7, 4) - (15/2, 5/2)$

$= \begin{pmatrix} -0.5 \\ 1.5 \end{pmatrix}$

so $T = \begin{pmatrix} 7 \\ 4 \end{pmatrix} + \begin{pmatrix} -0.5 \\ 1.5 \end{pmatrix}$

$= \begin{pmatrix} 6.5 \\ 5.5 \end{pmatrix}$ $T = (6.5, 5.5)$

Sub in $(6.5, 5.5)$ into $y = \frac{1}{3}x + k$ to find k

$y = \frac{1}{3}x + k$

$5.5 = \frac{1}{3} \times 6.5 + k$

$\frac{11}{2} = \frac{13}{6} + k$

$\boxed{k = \frac{10}{3}}$

## c)In more detail

**Way 1: Use discriminant**

Tangent means meets once hence discriminant $= 0$. First we need to build the equation (representing intersection) to use the discriminant on

$$y = \frac{1}{3}x + k$$

$$(x - 7)^2 + (y - 4)^2 = 2.5$$

We solve simultaneously

We can sub $y = \frac{1}{3}x + k$ into $(x - 7)^2 + (y - 4)^2 = 2.5$

$$(x - 7)^2 + \left(\frac{1}{3}x + k - 4\right)^2 = 2.5$$

$$x^2 - 14x + 49 + \frac{1}{9}x^2 + \frac{1}{3}kx - \frac{4}{3}x + \frac{1}{3}kx + k^2 - 4k - \frac{4}{3}x - 4k + 16 = 2.5$$

$$\frac{10}{9}x^2 + \left(-14 + \frac{1}{3}k - \frac{4}{3} + \frac{1}{3}k - \frac{4}{3}\right)x + k^2 - 4k - 4k + 49 + 16 - 2.5 = 0$$

$$\frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + k^2 - 8k + 62.5 = 0$$

Tangent $\Longrightarrow b^2 - 4ac = 0$ (tangent meets at one point hence one solution, so this is our hint to use the discriminant which talks about "how many" solutions)

$$\frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + (k^2 - 8k + 62.5) = 0$$

$$b^2 - 4ac = 0$$

$$\left[\frac{2}{3}k - \frac{50}{3}\right]^2 - 4\left(\frac{10}{9}\right)(k^2 - 8k + 62.5) = 0$$

$$\frac{4}{9}k^2 - \frac{200}{9}k + \frac{2500}{9} - \frac{40}{9}k^2 + \frac{320}{9}k - \frac{2500}{9} = 0$$

$$4k^2 - 200k + 2500 - 40k^2 + 320k - 2500 = 0$$

$$36k^2 - 120k = 0$$

$$4k(9k - 30) = 0$$

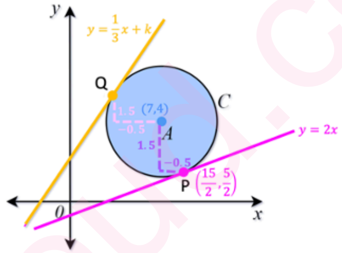
$$k = 0, k = \frac{30}{9} = \frac{10}{3}$$

$k = 0$ just picks up the $y = 2x$ tangent which we already know is the other tangent

$$k = \frac{10}{3}$$

**Way 2: Use symmetry**

**(use this if you have one of the points on the circumference and want to do the question quickly. We should use this if the tangent equation has a fraction in it since it saves a lot of time, but be aware we can't do this method if we don't have another point on the circumference)**




We need point P first. We solve the radius equation PA which is $y = -3x + 25$ and tangent equation $y = 2x$ simultaneously to get this point P.

$$y = -3x + 25$$

$$y = \frac{1}{3}x$$

$$\frac{1}{3}x = -3x + 25$$

$$\frac{10}{3}x = 25$$

$$x = \frac{15}{2}$$

When $x = \frac{15}{2}, y = \frac{1}{3}\left(\frac{15}{2}\right) = \frac{5}{2}$

To get from P to A:

$x: -0.5$
$y: +1.5$

So, to get from A to Q we do the same thing:

$x: -0.5$
$y: +1.5$

$$\binom{7}{4} + \binom{-0.5}{1.5} = \binom{6.5}{5.5}$$

So $Q(6.5, 5.5)$

We have equation $y = \frac{1}{3}x + k$

Plug this point (6.5, 5.5) in

$$5.5 = \frac{1}{3}(6.5) + k$$

$$5.5 = \frac{13}{6} + k$$

$$k = \frac{10}{3}$$

**16.** The curve $C$ has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and $a$ and $b$ are constants.

Given

- the point $(2, 10)$ lies on $C$
- the gradient of the curve at $(2, 10)$ is $-3$

(a) (i) show that the value of $a$ is $-2$

(ii) find the value of $b$.

**(4)**

(b) Hence show that $C$ has no stationary points.

**(3)**

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

**(2)**

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

**(2)**

a) differentiating f(x) eliminates b, so we can find the value of a. Differentiate f(x) by bringing down the power, and subtracting one from the power.

$f'(x) = 3ax^2 + 2\times15x - 39$

$f'(x) = 3ax^2 + 30x - 39$

@$x = 2$ $f'(x) = 3$

$3 = 3a\times2^2 + 30\times2 - 39$

$3 = 12a + 60 - 39$

$-24 = 12a$

$\boxed{a = -2}$

Now sub in $a=-2$, and $(2,10)$ into f(x) to find the value of b

$f(x) = ax^3 + 15x^2 - 39x + b$

$a = -2,\ x = 2,\ y = 10$

$10 = -2\times2^3 + 15\times2^2 - 39\times2 + b$

$10 = -16 + 60 - 78 + b$

$10 = -34 + b$

$\boxed{b = 44}$

**Question 16 continued**

b) Stationary points occur when the gradient $= 0$, $f'(x) = 0$. So we must show that there are no values of $x$ when $f'(x) = 0$

$$f'(x) = 3ax^2 + 30x - 39$$

$x = -2$ $\quad f'(x) = -6x^2 + 30x - 39$

set gradient $= 0$ $\rightarrow$ $0 = -6x^2 + 30x - 39$

use quadratic formula $\rightarrow$ $x = \dfrac{-30 \pm \sqrt{30^2 - (4 \times -6 \times -39)}}{2 \times -6}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-30 \pm \sqrt{900 - 936}}{-12}$$

$$= \frac{-30 \pm \sqrt{-36}}{-12}$$

the discriminant $(b^2 - 4ac)$ is negative. You cannot take the square root of a negative number, so there are no solutions of $x$.

$\boxed{f'(0) \text{ never equals } 0\text{, so } f(x) \text{ has no stationary point}}$

c)

| | $-2x^2$ | $7x$ | $-11$ |
|---|---|---|---|
| $x$ | 1. $-2x^3$ | 3. $7x^2$ | 5. $-11x$ |
| $-4$ | 2. $8x^2$ | 4. $-28x$ | 6. $44$ |

divide $-2x^3 + 15x^2 - 39x + 44$ by $(x-4)$ to find the quadratic Q.

$\boxed{f(x) = (x-4)(-2x^2 + 7x - 11)}$

d) When the curve intersects the $y$ axis, $x = 0$

$$f(0.2x) = f(0.2 \times 0) = f(0) = -2(0^3) + 15(0^2) - 39(0) + 44$$

$$f(0) = 44$$

so intersection with $y$ axis $= (0, 44)$

**Question 16 continued**

When the curve intersects the $x$ axis $y=0$, so $f(x)=0$

First find point of intersection of $f(x)$

$f(x)=0$

$$f(x)=-2x^3+15x^2-39x+44$$
$$0=(x-4)(-2x^2+7x-11)$$

$x=4$ $\quad x=\dfrac{-7\pm\sqrt{49-(4\times-2\times-11)}}{2\times-2}$

$=\dfrac{-7\pm\sqrt{-39}}{-4}$

use quadratic formula to solve for $x$

$(-2x^2+7x-11)$ gives no real solutions, so the only solution to $f(x)=0$ is $x=4$

when $f(x)=0$ $x=4$

intersection $=(4,0)$

We need to find when $f(0.2x)=0$. $f(0.2x)$ is a stretch of $f(x)$ parallel to the $x$ axis with a scale factor 5, so the point of intersection is $(4,0)$ multiplied by 5, which is $(20,0)$.

intersection of $x$ axis $(20,0)$
intersection of $y$ axis $(0,44)$

**(Total for Question 16 is 11 marks)**

**TOTAL FOR PAPER IS 100 MARKS**